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OCE3521

Homework 1

# Problem 1 – DD 2.2

## Part A

Starting with the material derivative for **u**

Most of the terms go to zero, leaving

The equation for the horizontal flow velocity is

Where Q is the volumetric flow rate and A(x) is the change in cross-sectional area along the x-axis. The equation for the changing cross-sectional area is

Plugging Equation 1.4 into 1.3

The acceleration of the fluid throughout the pipe is therefore

Plugging Equation 1.6 into 1.2

Solving for acceleration at x=0.5 m using Equation 1.7

## Part B

Beginning with Equation 1.3 and knowing the value of Q(t)

Calculating the derivative of Equation 1.9

Using Equation 1.10 to get acceleration at t=4.48 s and x=0.5 m

# Problem 2 – DD 2.3

## Part A

The velocity potential function can be broken down into its vector components

To find rotationality of the field, take the cross-product of the del operator and the velocity potential function

Since neither u or w have x or z variables, the cross-product is 0 and therefore the flow is irrotational

## Part B

Using the Equations 2.1-2.3, the flow divergence can be determined by

Since neither u or w have x or z variables, the dot-product is 0 and therefore the flow is non-divergent

## Part C

The stream function is defined by

Plugging in Equations 2.1 and 2.3 to 2.6

If the integration constant is assumed to be 0, the integral can be solved to

At t= T/8, the stream function is

For plotting the stream functions, two arbitrary constants, 0 and 1, were the value of

Rearranging Equations 2.10 and 2.11 in terms of z respectively yields

Which produces the following plot from x = [-10, 10]

Chart, line chart

Description automatically generated

Figure 1: Plot of two streamlines (Psi=0 and Psi=1)

# Problem 3 – DD 2.6

From momentum conservation in the z-axis, the following is known

Since the flow is in a steady state condition goes to zero. There are two additional accelerations acting on an inviscid, non-divergent flow: gravity and the pressure gradient. These can be derived from the body forces acting on the fluid

Removing *m* and substituting in Equation 3.1 yields

Rearranging Equation 3.3 yields the expected result

# Problem 4 – DD 2.7

The expansion begins by applying the del operator to the product of the three scalar functions

The Product Rule is used to take the respective derivative of each function product

Rearranging Equation 4.2 with common factors

The terms in parentheses can be simplified using the gradient operator yielding

# Problem 5 – DD 2.8

The velocity vector for the two-dimensional flow is given by

## Part A

The divergence of the flow is determined by the dot-product of the del operator and

Since the dot-product equals 0, the flow is non-divergent

## Part B

The rotationality of the flow is determined the cross-product of the del operator and

Since the resultant vector is 0, the flow is irrotational

## Part C

The streamline function is defined by Equation 2.6. When taken for this problem, the integral yields

When plotted, the streamline through points (1, 1) and (1, 2) produces the following graph

Chart

Description automatically generated

Figure 2: plot of Psi=0 and Psi=1 from Equation 5.4 through the points (1,1) and (1,2)

# Problem 6 – DD 2.10

## Part A

The first streamline for is

The second streamline for is

Which yields

When plotted, Equations 6.1 and 6.3 generate the following graph. Note, is a straight line at z = 0.

Chart

Description automatically generated

## Part B

When the streamline is and , the streamline equation reduces to

Rearranging to solve for z yields

To find where the derivative equals -5, the first derivative must be taken and solved for the x-value

Plugging in the x-value found from Equation 6.6 into 6.5 produces

Therefore, at (2, 5)

## Part C

From Equation 2.97 in *Water Wave Mechanics for Engineers and Scientists* by Dean and Dalrymple, the pressure gradient in a streamline can be determined by

By the definition of a streamline, and can be found with

Plugging in x=2, z=5, t=3, A=1, ρ=1 and simplifying Equations 6.9 and 6.10 into Equation 6.8 yields

# Problem 7 – DD 3.6

Starting with the definitions for the u and v components of the velocity potential, it is known that

Bernoulli’s equation is required to solve the problem and the u and v terms within can be replaced by Equations 7.1 and 7.2

Equation 7.3 can be simplified down to

The initial value problem for (1, 1) allows C(t) to be determined

Rearranging Equation 7.4 to solve for P yields

To find the location of local maxima and minima, the first derivative test must be performed

Since there is only one set of values, the absolute maximum value can be assumed to be at (0, 0). Therefore, plugging in (0, 0) to Equation 7.6 yields

# Appendix

## OCE3521\_Homework1\_Duffy.m

% OCE3521 - Homework 1

% Braidan Duffy

% Due: 02/11/21

%% Problem 2 - DD2.3

% Sketch two streamlines for t=T/8

% psi(t=T/8) = -cos(pi/4) \* (3z+5x)

x = -10:0.1:10;

z\_0 = -5/3 .\* x; % Z function when psi = 0

z\_1 = -1/3 \* (5.\*x+sec(pi/4)); % z function when psi = 1

% Plotting

figure(1)

title("DD2.3 - Sketch of two streamlines (Psi=0 and Psi=1)")

hold on

plot(x, z\_0)

plot(x, z\_1)

xlabel("X-Values")

ylabel("Z-Values")

legend("Psi=0", "Psi=1")

hold off

%% Problem 5 - DD2.8

% Sketch the two streamlines through (1, 1) and (1, 2)

x = -10:0.1:10;

z1 = atan(1) ./ x;

z2 = atan(0.5) ./ x;

% Plotting

figure(2)

title("DD2.8 - Sketch the stream lines through (1,1) and (1,2)")

hold on

plot(x, z1)

plot(x, z2)

xlabel("X-Values")

ylabel("Z-Values")

legend("Psi=0", "Psi=1")

hold off

%% Problem 6 - DD2.10

% Sketch the streamlines for psi=0 and psi=6A

x = 0:0.1:10; % Domain x = [0, 10]

z\_2 = 2 ./ x .^ 2; % Implicit z function for

% Plotting

figure(3)

title("DD2.10 - Sketch the streamlines for Psi=0 and Psi=6A")

hold on

plot(x, zeros(1, length(x))) % first function of z is implicitly 0

plot(x, z\_2)

xlabel("X-Values")

ylabel("Z-Values")

legend("Psi=0", "Psi=6A")

hold off